



# Combined free and forced convection laminar film condensation on an inclined circular tube with isothermal surface

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## Abstract

Laminar film condensation on an inclined circular tube, under the condition of combined free and forced convection, is analyzed. The assumptions are as in the analysis of Shekrladze and Gomelaury [Theoretical study of laminar film condensation of flowing vapour, *Int. J. Heat Mass Transfer* 9 (1966) 581–591]. In addition, some approximations are introduced for the determination of the interfacial shear stress. The resultant governing equation, in special cases, yields the known analytical solutions of horizontal and vertical tubes, which were obtained in previous studies. A numerically obtained solution reveals the effects of vapour velocity and gravity forces on local and mean Nusselt numbers. For the case of an inclined tube with infinite length, an explicit simple expression has been obtained, based on numerical results, to calculate the mean Nusselt number for the whole tube surface. © 1999 Elsevier Science Ltd. All rights reserved.

## 1. Introduction

The problem of forced convection laminar film condensation, under the combined effect of gravity force and vapour velocity, has received considerable attention in recent studies.

Shekrladze and Gomelaury [1] solved this problem numerically for a horizontal tube in a vertical vapour downflow. They used an approximate expression for the interfacial shear stress, together with the assumptions of Nusselt [2]. Their numerical results were shown by Rose [3] to be represented to within 4% by

$$\overline{Nu}_d / \sqrt{\tilde{Re}_d} = \frac{0.9 + 0.728F_0^{1/2}}{(1 + 3.44F_0^{1/2} + F_0)^{1/4}} \quad (1)$$

where  $\overline{Nu}_d$  is the mean Nusselt number,  $\tilde{Re}_d$  is the two-phase Reynolds number and  $F_0$  is a dimensionless par-

ameter which measures the relative importance of gravity force and vapour velocity on the condensate film motion.

A more accurate model for the interfacial shear stress was made in the analysis of Fujii et al. [4], where the solution of the condensate film and vapour boundary layer equations was made with matching of the shear stress at the interface. Inertia, convection and pressure gradient effects were also neglected in the analysis. Although their analysis enables the vapour boundary layer separation to be determined, however, it gives values of  $\overline{Nu}_d$  that are in close agreement with that calculated from Eq. (1), except at a low condensation rate condition [5,6].

Krupiezka [7] investigated the effect of surface tension, due to the curvature of the condensate film on a horizontal tube, in a Nusselt type model. He concluded

### Nomenclature

$C_p$	specific heat of condensate at constant pressure
$d$	tube diameter
$F_0$	$= (Gr_d / \tilde{Re}_d^2)(\mu h_{fg}'' / (k\Delta T))$ , dimensionless parameter
$F_d$	$= F_0 / \cos \varphi$ , dimensionless parameter
$F_z$	$= (Gr_z / \tilde{Re}_z^2)(\mu h_{fg}'' / (k\Delta T))$ , dimensionless parameter
$g$	acceleration of gravity
$Gr_d$	$= g\rho^2 d^3 / \mu^2$ , Grashof number for tube
$Gr_z$	$= g\rho^2 z^3 / \mu^2$ , Grashof number for vertical plate
$h_{fg}$	latent heat of condensation
$h_{fg}''$	modified latent heat of condensation
$k$	thermal conductivity of condensate
$L$	tube length
$L^+$	$= 2L / (d \tan \varphi)$ , dimensionless tube length
$\dot{m}$	condensate mass flow rate
$\overline{m_c''}$	condensation mass flux
$\overline{Nu}_d$	$= \bar{\alpha} d / k$ , mean Nusselt number
$\overline{Nu}_d(z)$	$= \alpha_z d / k$ , peripherally-averaged local Nusselt number
$\tilde{Re}_d$	$= V_\infty d / \nu$ , two-phase Reynolds number for tube
$\tilde{Re}_z$	$= V_\infty z / \nu$ , two-phase Reynolds number for vertical tube
$u, w$	condensation velocity components in $x$ - and $z$ -directions, respectively
$V_\infty$	free-stream vapour velocity

$V_x$	$x$ -component of vapour velocity at the edge of vapour boundary layer
$V_z$	$z$ -component of vapour velocity at the edge of vapour boundary layer
$x$	peripheral coordinate
$z$	axial coordinate
$Z^+$	dimensionless axial coordinate

### Greek symbols

$\alpha$	$= k / \delta$ , local heat transfer coefficient
$\delta$	local film thickness
$\Delta$	dimensionless local film thickness, Eq. (15)
$\Delta T$	$= (T_s - T_w)$ , temperature drop across the condensate film
$\eta$	dimensionless local film thickness, Eq. (22)
$\mu$	dynamic viscosity
$\nu$	kinematic viscosity of condensate
$\rho$	density of condensate
$\tau_{\delta x}$	interfacial vapour shear in $x$ -direction
$\tau_{\delta z}$	interfacial vapour shear in $z$ -direction
$\phi$	peripheral angle from top point of tube
$\varphi$	angle of inclination of tube with horizontal

### Subscripts

$s$	saturation
$w$	wall
$x$	$x$ -direction
$z$	$z$ -direction

that the surface tension effect is only significant for small-diameter tubes.

Rose [3] using the assumptions of Shekrladze and Gomelaury examined the effect of pressure gradient in the condensate film. Rose concluded that for  $(\rho_\nu V_\infty^2) / (g\rho d) \leq 1/8$ , where a solution could be obtained for the whole surface, the mean Nusselt number is within 1% of that found when ignoring the pressure term.

A recent state-of-the-art review made by Rose [8] summarized that inertia and convection effects in forced convection film condensation can, for most practical circumstances, be generally neglected.

Concerning combined convection film condensation on inclined circular tubes encountered in many practical applications, no theoretical studies (to the author's knowledge) are available in literature. Therefore, the aim of the present work is to analyze this problem of mixed-convection film condensation outside an inclined circular tube. The analysis will be performed based on the assumptions of Nusselt, together with adopting the vapour shear model of Shekrladze and Gomelaury.

## 2. Analysis and mathematical formulation

The physical model and coordinate system used are shown in Fig. 1. A pure, dry saturated vapour with temperature  $T_s$  flows downward (with uniform 'free stream' velocity  $V_\infty$ ) over a circular tube inclined with angle  $\varphi$  to the horizontal. This tube is cooled internally such that the wall temperature  $T_w$  is uniform and much lower than the vapour saturation temperature  $T_s$ . Thus, a continuous condensate film will form outside the tube flowing in both the axial and peripheral directions. The assumptions employed in the formulation of the problem are:

1. Condensate film thickness is much smaller than the tube diameter.
2. The inertia and pressure terms in the momentum equation and the convection terms in the energy equation for the condensate film can be neglected.
3. Surface tension effect is insignificant.
4. The condensate film flow is laminar, steady and

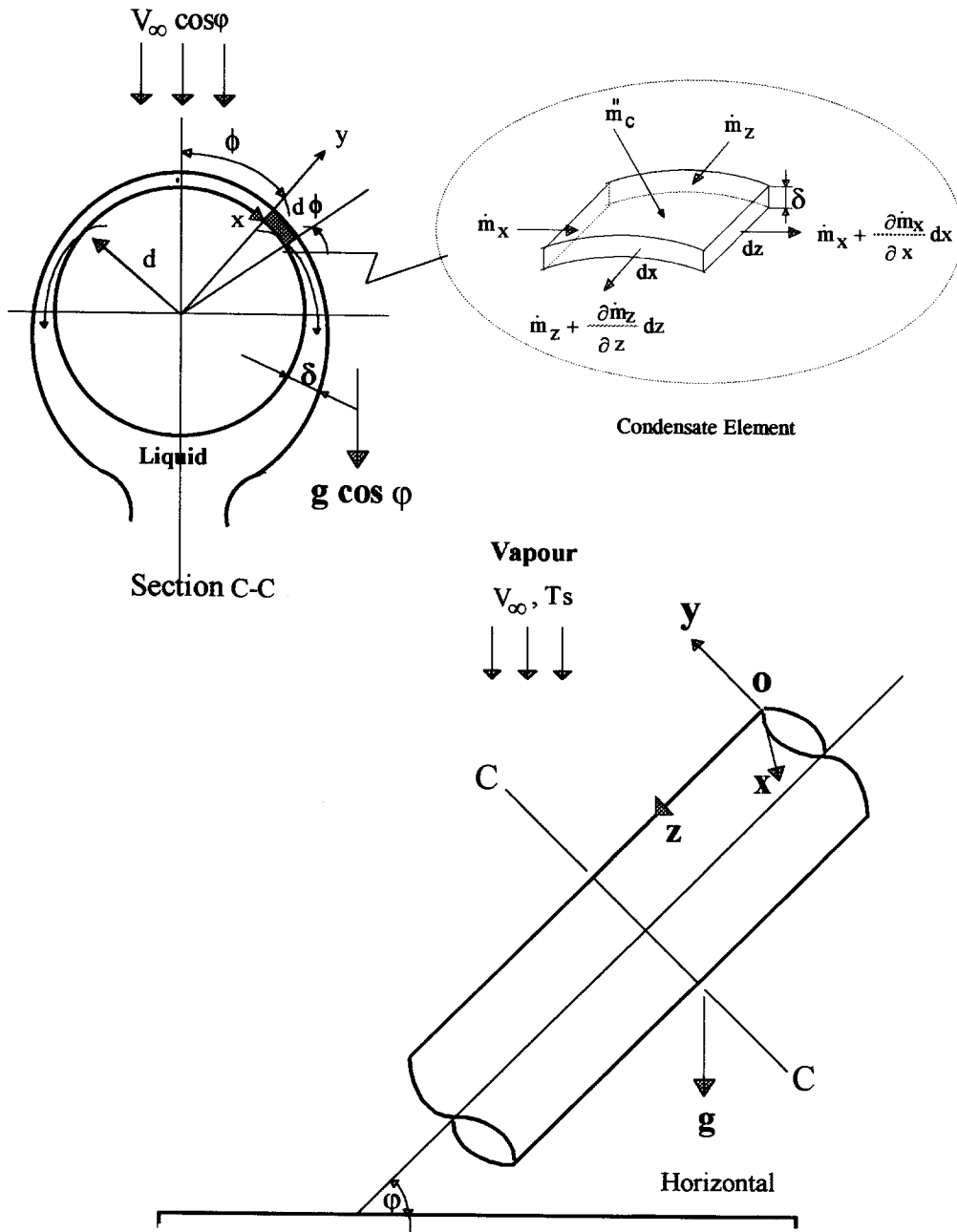


Fig. 1. Physical model.

with negligible viscous dissipation.

5. All physical properties of the condensate film are constant.

Based on these assumption, the  $x$ - and  $z$ -momentum equations for the condensate film are written, respectively, as

$$\mu \frac{\partial^2 u}{\partial y^2} + \rho g \sin \phi \cos \phi = 0 \tag{2a}$$

$$\mu \frac{\partial^2 w}{\partial y^2} + \rho g \sin \phi = 0 \tag{2b}$$

with the boundary and interface conditions

$$\text{at } y = 0, \quad u = 0, \quad w = 0 \quad (3)$$

$$\text{at } y = \delta, \quad \frac{\partial u}{\partial y} = \frac{\tau_{\delta x}}{\mu}, \quad \frac{\partial w}{\partial y} = \frac{\tau_{\delta z}}{\mu} \quad (4)$$

wherein  $u$  and  $w$  are, respectively, the velocity components in the  $x$ - and  $z$ -directions. The symbols  $\tau_{\delta x}$  and  $\tau_{\delta z}$  denote, respectively, to the  $x$ - and  $z$ -components of interfacial shear stress.

Integrating Eqs. (2a) and (2b) with the use of boundary conditions (3) and (4) gives, respectively,

$$u = \frac{1}{\mu} \{ y \tau_{\delta x} - \rho g \cos \varphi \sin \phi (y^2/2 - \delta y) \} \quad (5)$$

$$w = \frac{1}{\mu} \{ y \tau_{\delta z} - \rho g \sin \varphi (y^2/2 - \delta y) \} \quad (6)$$

Considering potential vapour flow with uniform vertical velocity  $V_\infty$ , the vapour velocity at the edge of the boundary layer may be derived in the  $x$ - and  $z$ -directions, respectively, by

$$V_x = 2V_\infty \cos \varphi \sin \phi \quad (7)$$

$$V_z = V_\infty \sin \varphi \quad (8)$$

In a similar way to that made by Shekrladze and Gomelaury [1], the interfacial shear stress can be evaluated (under infinite-condensation-rate condition) in the  $x$ - and  $z$ -directions by:

$$\tau_{\delta x} = m_c''(V_x - u_\delta) \approx m_c'' V_x \quad (9)$$

$$\tau_{\delta z} = m_c''(V_z - w_\delta) \approx m_c'' V_z \quad (10)$$

wherein  $m_c''$  is the local condensation mass flux. Eqs. (9) and (10) require that  $V_x \gg u_\delta$  and  $V_z \gg w_\delta$ . Investigating Eqs. (7) and (9) reveals that the peripheral interfacial shear stress  $\tau_{\delta x}$  is positive for all values of the peripheral angle  $\phi$ . Thus, the possibility of vapour boundary layer separation is excluded from this analysis.

A mass balance for the condensate element, shown in Fig. 1, yields

$$m_c'' = \delta \left( \frac{\partial \dot{m}_x}{\partial x} + \frac{\partial \dot{m}_z}{\partial z} \right) = \rho \frac{\partial}{\partial x} \left( \int_0^\delta u \, dy \right) + \rho \frac{\partial}{\partial z} \left( \int_0^\delta w \, dy \right) \quad (11)$$

and a heat balance at the condensate-vapour interface, as in Nusselt's theory [2], gives

$$m_c'' = \frac{k \Delta T}{h_{fg}'' \delta} \quad (12)$$

where  $h_{fg}'' = h_{fg} + 3C_p \Delta T/8$  is the modified latent heat of condensation proposed by Rohsenow [9] to account for the effects of heat convection in the film.

When Eqs. (7)–(10) are used to eliminate  $\tau_{\delta x}$  and  $\tau_{\delta z}$  from Eqs. (5) and (6), respectively, the integral in Eq. (11) may be evaluated and the resultant value of  $m_c''$  can be used in Eq. (12). With some rearrangement one gets the following differential equation:

$$\begin{aligned} \cos \varphi \sin \phi \left( \frac{\tilde{R}e_d}{d^2} + \frac{\mu h_{fg}''}{k \Delta T} \cdot \frac{Gr_d}{d^4} \delta^2 \right) \frac{\partial \delta^2}{\partial \phi} + \frac{d}{4} \sin \varphi \left( \frac{\tilde{R}e_d}{d^2} + 2 \frac{\mu h_{fg}''}{k \Delta T} \cdot \frac{Gr_d}{d^4} \delta^2 \right) \frac{\partial \delta^2}{\partial z} + \frac{2}{3} \cos \varphi \cos \phi \left( 3 \frac{\tilde{R}e_d}{d^2} + \frac{\mu h_{fg}''}{k \Delta T} \cdot \frac{Gr_d}{d^4} \delta^2 \right) \delta^2 = 1 \end{aligned} \quad (13)$$

for the local condensate film thickness  $\delta$  as a function of the two-phase Reynolds number  $\tilde{R}e_d$ , Grashof number  $Gr_d$ , inclination angle  $\varphi$ , peripheral angle  $\phi$  and axial location  $z$ .

Eq. (13) can be rewritten in the dimensionless form:

$$\begin{aligned} \sin \phi (1 + 4F_d \Delta^2) \frac{\partial \Delta^2}{\partial \phi} + \frac{1}{2} (1 + 8F_d \Delta^2) \frac{\partial \Delta^2}{\partial Z^+} + 2 \cos \phi \left( 1 + \frac{4}{3} F_d \Delta^2 \right) \Delta^2 = 1/4 \end{aligned} \quad (14)$$

where

$$\Delta = \frac{\delta}{2d} \sqrt{\tilde{R}e_d \cos \varphi} \quad (15)$$

$$Z^+ = 2z/(d \tan \varphi) \quad (16)$$

and

$$F_d = (Gr_d/(\tilde{R}e_d^2 \cos \varphi)) (\mu h_{fg}''/(k \Delta T)) \quad (17)$$

The boundary conditions are:

$$\text{at } Z^+ = 0; \quad \Delta = 0; \quad \pi \geq \phi \geq 0 \quad (18)$$

$$\text{at } \phi = 0, \quad \phi = \pi; \quad \frac{\partial \Delta}{\partial \phi} = 0; \quad \infty \geq Z^+ \geq 0 \quad (19)$$

$$\text{at } Z^+ \rightarrow \infty; \quad \frac{\partial \Delta}{\partial Z^+} = 0 \quad (20)$$

The boundary condition (18) indicates that the film thickness is zero around the tube at its upper end. The second boundary condition expresses the symmetry of the condensate film and its smoothness at the top

( $\varphi=0$ ) and bottom ( $\varphi=\pi$ ) points along the tube. The third condition implies that for a large distance from the upper tube end, the condensate film thickness varies only around the tube. The same condition was used in previous studies made on the problem of free convection film condensation on inclined circular and elliptical tubes [10,11].

### 3. Solution

#### 3.1. Special solutions

Before solving Eqs. (13) or (14), which represent the general case, the following special cases will be considered first.

##### 3.1.1. Free convection film condensation on an inclined tube

For a quiescent vapour ( $V_\infty=0$  or  $\tilde{Re}_d=0$ ), Eq. (13) reduces to one which can be reformulated as

$$\frac{\partial \eta}{\partial Z^+} + \sin \phi \frac{\partial \eta}{\partial \phi} = 4/3(1 - \eta \cos \phi) \tag{21}$$

for the dimensionless film thickness,

$$\eta = \frac{2\delta^4}{3d^4} Gr_d \frac{\mu h'_{fg}}{k\Delta T} \cos \phi \tag{22}$$

Eq. (21) is the same result found originally by Hassan and Jakob [10]. Later, Mosaad [12] found the same governing equation (21) by analyzing the problem in the framework of boundary layer theory.

For the case of an inclined tube with infinite-length, the term  $\partial \eta / \partial Z^+$  may be neglected, as made in the analysis of Hassan and Jakob [10] and that of Fieg and Roetzel [11]. Hence, the solution of the reduced equation; for  $\partial \eta / \partial \phi = 0$  at  $\phi=0$ , leads finally to the mean Nusselt number formula:

$$\overline{Nu}_d = 0.728 \left\{ Gr_d \frac{\mu h'_{fg}}{k\Delta T} \cos \phi \right\}^{1/4} \tag{23}$$

which is the same empirical formula proposed by Selin [13] to achieve good agreement with his own experimental data. This relation for  $\varphi=0$  simplifies to the known Nusselt's expression of the horizontal tube.

Eq. (23) can be rewritten in the alternative form:

$$\overline{Nu}_d / \sqrt{\tilde{Re}_d \cos \phi} = 0.728 F_d^{1/4} \tag{24}$$

##### 3.1.2. Forced convection dominated film condensation on an inclined tube

For  $g = 0$  (i.e.,  $F_d=0$ ), Eq. (14) simplifies to

$$\frac{\partial \Delta^2}{\partial Z^+} + 2 \sin \phi \frac{\partial \Delta^2}{\partial \phi} = 1/2 - 4\Delta^2 \cos \phi \tag{25}$$

which is the same result found by Mosaad [14].

For the case of the infinite-tube-length, the term  $\partial \eta / \partial Z^+$  may be neglected. Hence, Eq. (25) simplifies to one, which leads finally to the mean Nusselt number expression:

$$\overline{Nu}_d / \sqrt{\tilde{Re}_d \cos \phi} = 0.9 \tag{26}$$

which for  $\varphi=0$  (horizontal tube) simplifies to that of Shekrladze and Gomelaury [1].

##### 3.1.3. Combined free and forced convection film condensation on a horizontal tube

For  $\varphi=0$  in Eq. (13), one gets, after some rearrangement

$$\begin{aligned} \sin \phi (1 + 4F_0 \Delta_0^2) \frac{\partial \Delta_0^2}{\partial \phi} \\ + 2 \cos \phi \left( 1 + \frac{4}{3} F_0 \Delta_0^2 \right) \dots \Delta_0^2 = 1/4 \end{aligned} \tag{27}$$

where  $\Delta_0$  is a dimensionless local film thickness defined by Eq. (15) for  $\varphi=0$ . Eq. (27) is the same result found by Shekrladze and Gomelaury [1], which is solved numerically. Their numerically obtained values of the mean Nusselt number  $\overline{Nu}_d$  were shown by Rose [3] to be represented to within 0.4% by Eq. (1).

##### 3.1.4. Combined free and forced convection film condensation on a vertical tube

For  $\varphi=\pi/2$  in Eq. (13) omitting the  $\phi$ -derivative terms, one gets after some rearrangement:

$$(1 + 8F_0 \Delta_0^2) \frac{\partial \Delta_0^2}{\partial Z^+} = 1/2 \tag{28}$$

where  $\Delta_0$ ,  $Z^+$  and  $F_0$  are, respectively, defined by Eqs. (15)–(17) omitting the  $\varphi$ -terms involved.

Solving Eq. (28); for  $\Delta_0=0$  at  $Z^+=0$ , finally leads to the expression:

$$Nu_z / \sqrt{\tilde{Re}_z} = \frac{1}{2\sqrt{2}} \sqrt{1 + \sqrt{1 + 16F_z}} \tag{29}$$

for the local Nusselt number  $Nu_z$  as a function of  $\tilde{Re}_z$  and  $F_z$ .

Shekrladze and Gomelaury [1] found Eq. (29) previously and recommended applying it for the vertical tube provided that  $\delta \ll d$ .

So far, it is clear from the above-mentioned special cases that the model yields the same solutions obtained in previous studies. This proves its validity and correctness.

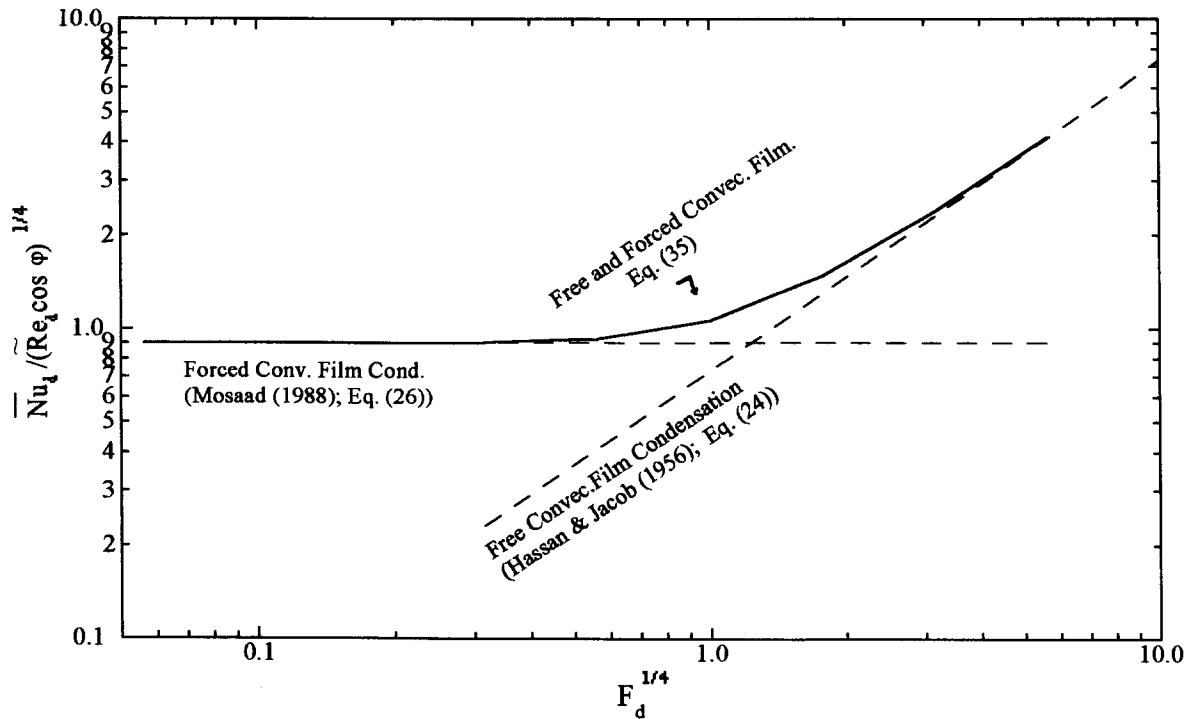


Fig. 2. Mean Nusselt number for infinitely long tube as a function of  $F_d$ .

3.2. General solution

First, the simple limiting case of an inclined tube with infinite length is considered. Then, the more general case of the finite-tube-length will be dealt with.

3.2.1. Infinite-length inclined tube

For this case, the term  $\partial\eta/\partial Z^+$  in the general governing equation (14) may be neglected. Hence, this equation simplifies to

$$\sin \phi (1 + 4F_d \Delta^2) \frac{\partial \Delta^2}{\partial \phi} + 2 \cos \phi \left( 1 + \frac{4}{3} F_d \Delta^2 \right) \Delta^2 = 1/4 \tag{30}$$

Fieg and Roetzel [11] found an approximate solution for the problem of free convection film condensation on an infinite-length inclined elliptical tube by neglecting the axial variation in the film thickness compared to the peripheral variation. In addition, the finite-tube solution presented in the next subsection reveals that beyond a short length from the upper tube end of  $Z^+ \approx 2.6$ , the  $z$ -variation of film thickness is negligible. Thus, this approximation of neglecting the term  $\partial\eta/\partial Z^+$  in the solution of the infinite-length case will be of negligible influence on the calculated mean Nusselt number (cf, Fig. 4).

The above Eq. (30), with  $\Delta$  and  $F_d$  defined, respectively, from Eqs. (15) and (17) for  $\cos \phi = 1$ , reduces to the same Eq. (27) of the horizontal tube.

Eq. (30) with boundary condition (19) can be solved numerically to calculate  $\Delta(\phi)$  around the tube; for different values of  $F_d$ . These numerically obtained values of  $\Delta(\phi)$  can be used to calculate the local Nusselt number from

$$Nu_\phi / \sqrt{\tilde{R}e_d \cos \phi} = \frac{1}{2\Delta(\phi)} \tag{31}$$

and the mean Nusselt number from

$$\bar{Nu}_d / \sqrt{\tilde{R}e_d \cos \phi} = \frac{1}{\pi} \int_0^\pi \frac{1}{2\Delta(\phi)} d\phi \tag{32}$$

Employing a fourth-order Runge–Kutta numerical integration procedure performed the numerical solution of Eq. (30). Step size of  $\Delta\phi = 0.05^\circ$  was used. Simpson’s rule has been used to calculate numerically the integration in Eq. (32). The upper limit of this integration was limited to  $\phi = 3.125$  instead of  $\phi = \pi$ . This is to overcome the problem that the value of  $\Delta$ , calculated numerically from Eq. (30), goes to infinity as  $\phi = \pi$ .

3.2.2. Finite-length inclined tube

The case of the finite-length tube represents the most

general case. For this case, the main governing equation (14) has to be solved numerically subject to boundary conditions (18)-(20). This is to calculate the dimensionless local film thickness  $\Delta$  as a function of  $F_d$ . Then, the peripherally averaged local Nusselt number can be calculated from

$$\overline{Nu}_d(z)/\sqrt{\tilde{Re}_d \cos \varphi} = \frac{1}{\pi} \int_0^\pi \frac{1}{2\Delta(\phi, Z^+)} d\phi \quad (33)$$

and the mean Nusselt number from

$$\overline{Nu}_d/\sqrt{\tilde{Re}_d \cos \varphi} = \frac{1}{\pi L^+} \int_0^{L^+} \int_0^\pi \frac{d\phi}{2\Delta(\phi, Z^+)} dZ^+ \quad (34)$$

where  $L^+$  is the dimensionless total tube length.

The same numerical techniques used in the above section, have also been used to solve Eq. (14) as well as to calculate the integration of Eqs. (33) and (34). Step sizes of  $\Delta\phi=0.05$  and  $\Delta Z^+=0.1$  were used in solving Eq. (14). To assess the accuracy of these numerical solutions, the analytical solutions (24) and (26) were used as a reference. The relative deviations between the analytically and numerically calculated values of mean Nusselt numbers were around  $\pm 1\%$ .

#### 4. Results and discussion

For the case of an inclined tube with infinite-length, the values of the normalized mean Nusselt number,

$$\overline{Nu}_d/\sqrt{\tilde{Re}_d \cos \varphi},$$

calculated numerically from Eq. (32) for different values of  $F_d$  ranging from 0-10<sup>4</sup>, were found to be represented with an accuracy of  $\pm 0.5\%$  by

$$\overline{Nu}_d/\sqrt{\tilde{Re}_d \cos \varphi} = \frac{0.9 + 0.728F_d^{1/2}}{(1 + 3.44F_d^{1/2} + F_d)^{1/4}} \quad (35)$$

The similarity between Eqs. (27) and (30) explains that between Eqs. (1) and (35).

Fig. 2 displays the above relation in terms of

$$\overline{Nu}_d/\sqrt{\tilde{Re}_d \cos \varphi}$$

versus  $F_d^{1/4}$ . It is seen that as  $F_d$  goes to infinity (i.e.,  $\tilde{Re}_d \rightarrow 0$ ), Eq. (35) satisfies the free convection (quiescent vapour) film condensation solution (24) found previously by Hassan and Jakob [10]. For the other extreme of  $F_d \rightarrow 0$  (i.e.,  $g = 0$  or  $\tilde{Re}_d \rightarrow \infty$ ), Eq. (35) satisfies the forced-convection-dominated laminar film condensation solution (26) obtained previously by Mosaad [14].

The case of the finite-length tube, numerical results,

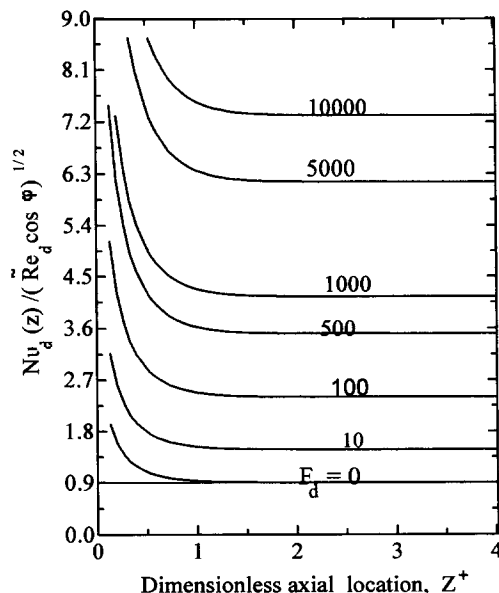


Fig. 3. Peripherally-averaged local Nusselt number along a finite-length tube for various values of  $F_d$ .

calculated from Eqs. (33) and (34), are plotted in Figs. 3 and 4, respectively. The distribution of the peripherally averaged Nusselt number along the tube is displayed in Fig. 3 in terms of

$$Nu_d(z)/\sqrt{\tilde{Re}_d \cos \varphi}$$

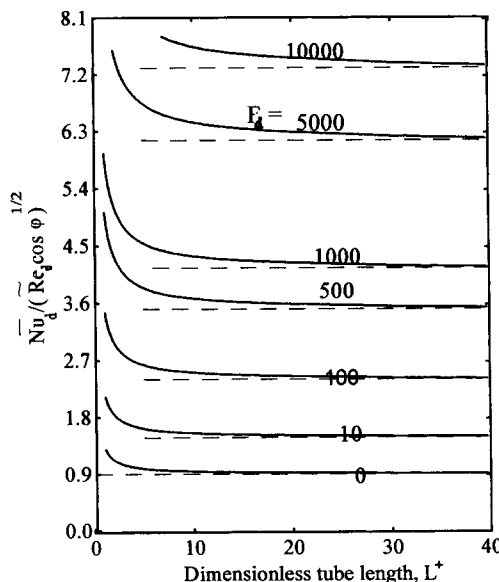


Fig. 4. Mean Nusselt number as a function of the total tube length, for different values of  $F_d$ ; dashed lines represent the infinite-length tube solution given by Eq. (35).

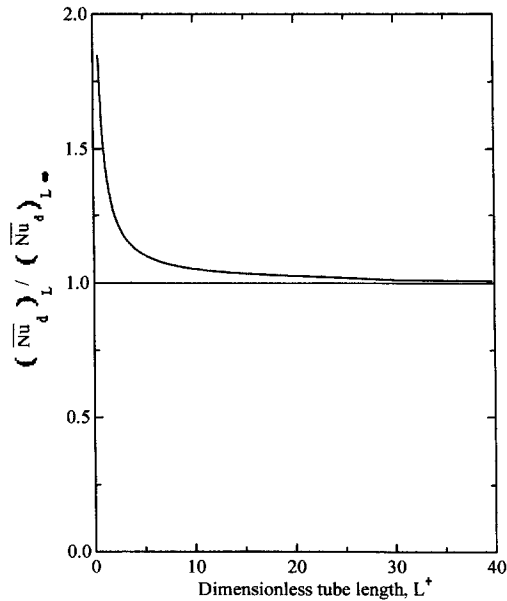


Fig. 5. Ratio of the mean Nusselt numbers of finite- and infinite-length inclined tubes vs the dimensionless total tube length.

versus  $Z^+$ , for different values of  $F_d$ . At fixed  $Z^+$ ,  $Nu_d(z)$  increases with increasing  $F_d$ . However, for constant  $F_d$ ,  $Nu_d(z)$  decreases from an infinite value at the start point ( $Z^+=0$ ) with increasing  $Z^+$  to assume a constant value as  $Z^+ \rightarrow \infty$ , or more precisely as  $Z^+$  exceeds the value 2.6. This constant value is 0.9 for  $F_d=0$ , i.e., the same solution of forced convection dominated film condensation [cf. Eq. (26)].

Fig. 4 shows the dependence of the mean Nusselt number for the whole tube surface  $\bar{Nu}_d$  [calculated from Eq. (34)] on the dimensionless total tube length  $L^+$  and the dimensionless parameter  $F_d$ . It is clear that the term

$$\bar{Nu}_d / \sqrt{\tilde{Re}_d \cos \varphi}$$

decreases with an increase in the total tube length  $L^+$ . As  $L^+$  exceeds 40, this term assumes the constant infinite-length values [calculated by Eq. (35) as a function of  $F_d$ ]. Dashed lines in the plot represent the infinite-tube solution (35). Therefore, these results of Fig. 3 could be represented by one curve in Fig. 5, in terms of ratio of the mean Nusselt number of the finite-length tube to that of the infinite-length tube versus the dimensionless total tube length  $L^+$ .

## 5. Conclusions

The heat transfer problem of forced convection laminar film condensation on an inclined tube has been analyzed with considering the combined influence of vapour shear and gravity forces. The vapour shear at the condensate surface was modelled by assuming potential vapour flow outside the vapour boundary layer, together with employing the infinite-condensation-rate approximation of Shekrladze and Gomelaouri [1].

For free convection (quiescent vapour) laminar film condensation ( $V_\infty=0$  or  $F_d \rightarrow \infty$ ), the model yields the same result of Hassan and Jakob [10]. The more special case of the infinite-length tube, expression (23) for calculating the mean Nusselt number for the whole tube surface  $\bar{Nu}_d$  has been derived analytically, which is the same empirical correlation proposed by Selin [13] based on his own experimental data.

For forced-convection-dominated film condensation ( $F_d \rightarrow 0$ ), the model gives the same result as Mosaad [14]. Additionally, the model yields, in special cases, the known analytical solutions of horizontal ( $\varphi=0$ ) and vertical ( $\varphi=\pi/2$ ) tubes, which were obtained previously by other authors. This proves the validity of the present approach.

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